Graphing Equations

Britannica® Mathematics in Context

Algebra



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Opinions expressed are those of the authors and not necessarily those of the Foundation.

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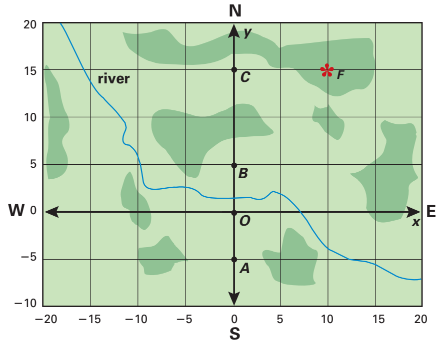
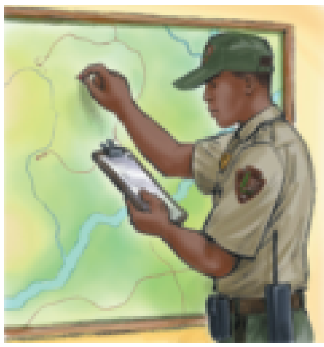
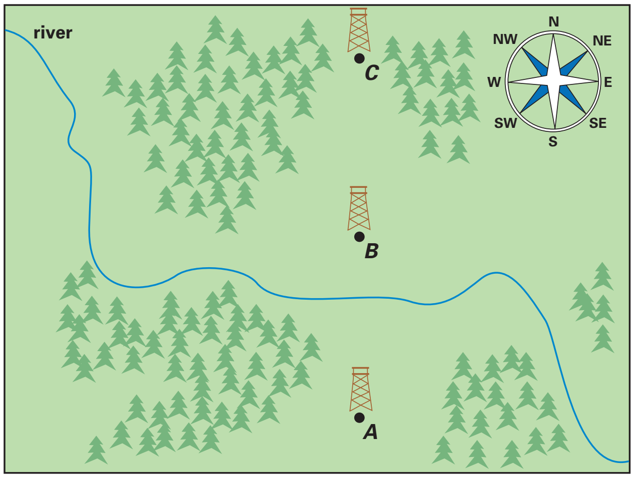




Dear Student,

Graphing Equations is about the study of lines and solving equations. At first you will investigate how park rangers at observation towers report forest fires. You will learn many different ways to describe directions, lines, and locations. As you study the unit, look around you for uses of lines and coordinates in your day-to-day activities.



You will use equations and inequalities as a compact way to describe lines and regions.

A "frog" will help you solve equations by jumping on a number line. You will learn that some equations can also be solved by drawing the lines they represent and finding out where they intersect.

We hope you will enjoy this unit.

Sincerely,

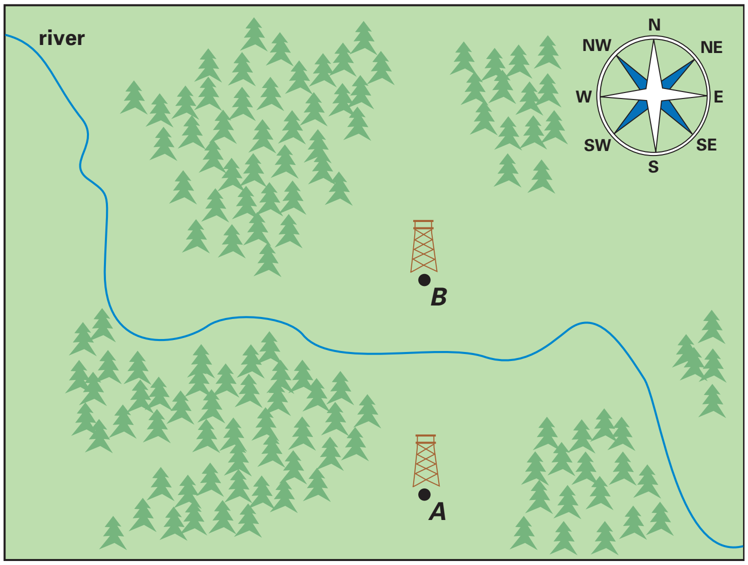
The Mathematics in Context Development Team

A Where There's Smoke

Where's the Fire?

From tall fire towers, forest rangers watch for smoke. To fight a fire, firefighters need to know the exact location of the fire and whether it is spreading. Forest rangers watching fires are in constant telephone communication with the firefighters.

The map shows two fire towers at points *A* and *B*. The eight-pointed star in the upper right corner of the map, called a **compass rose**, shows eight directions: north, northeast, east, southeast, south, southwest, west, and northwest. The two towers are 10 kilometers (km) apart, and as the compass rose indicates, they lie on a north-south line.



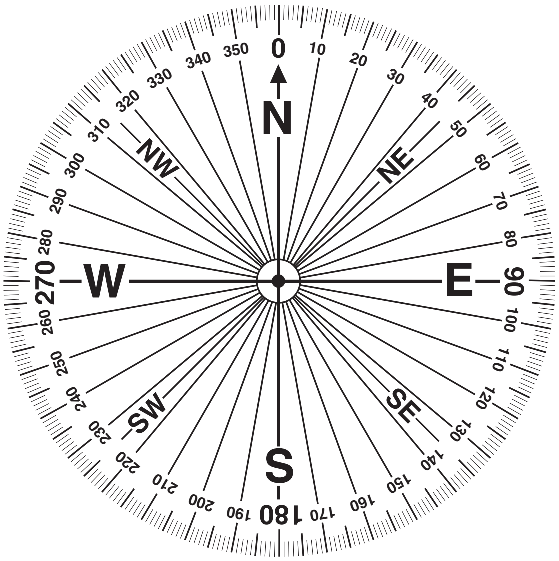
One day the rangers at both fire towers observe smoke in the forest.

The rangers at tower *A* report that the smoke is directly northwest of their tower.

1. Is this information enough to tell the firefighters the exact location of the fire? Explain why or why not.

The rangers at tower *B* report that the smoke is directly southwest of their tower.

2. Use **Student Activity Sheet 1** to indicate the location of the fire.



In problems 1 and 2, you used the eight points of a compass rose to describe directions. You can also use **degree measurements** to describe directions.

A complete circle contains 360°. North is typically aligned with 0° (or 360°). Continuing in a clockwise direction, notice that east corresponds with 90°, south with 180°, and west with 270°.

You measure directions in degrees, clockwise, starting at north.

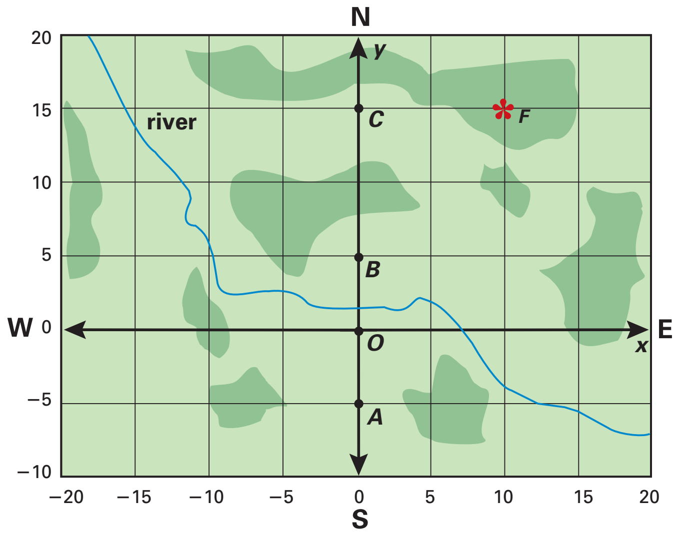
Smoke is reported at 8° from tower *A*, and the same smoke is reported at 26° from tower *B*.

3. Use **Student Activity Sheet 2** to show the exact location of the fire.

4. Use **Student Activity Sheet 2** to show the exact location of a fire if rangers report smoke at 342° from tower *A* and 315° from tower *B*.

Coordinates on a Screen

The park supervisor uses a computerized map of the National Park to record and monitor activities in the park. He also uses it to locate fires.



The computer screen on the left shows a map of the National Park. The shaded areas indicate woods. The plain areas indicate meadows and fields without trees. The numbers represent distances in kilometers.

Point *O* on the screen represents the location of the park supervisor's office, and points *A*, *B*, and *C* are the rangers' towers.

5. **a.** What is the distance between towers *A* and *B*? Between tower *C* and point *O*?

   b. How is point *O* related to the positions of towers *A* and *B*?

A fire is spotted 10 km east of point *C*. The location of that point (labeled *F*) is given by the coordinates 10 and 15. The coordinates of a point can be called the **horizontal coordinate** and the **vertical coordinate**, or they can be called the ***x*-coordinate** and the ***y*-coordinate**, depending on the variables used in the situation.

$$F = (10, 15)$$

horizontal coordinate or *x*- coordinate

vertical coordinate or *y*- coordinate

Use the map on page 3 to answer problems 6 and 7.

6. a. Find the point that is halfway between *C* and *F*. What are the coordinates of that point?

   b. Write the coordinates of the point that is 10 km west of *B*.

The coordinates of fire tower *B* are (0, 5).

7. a. What are the coordinates of the fire towers at *C* and at *A*?

   b. What are the coordinates of the office at *O*?

The rangers' map is an example of a **coordinate system**. Point *O* is called the **origin** of the coordinate system. If the coordinates are written as (*x*, *y*):

the horizontal line through *O* is called the ***x*-axis**.

the vertical line through *O* is called the ***y*-axis**.

The two axes divide the screen into four parts: a northeast (NE) section, a northwest (NW) section, a southwest (SW) section, and a southeast (SE) section. Point *O* is a corner of each section, and the sections are called **quadrants**.

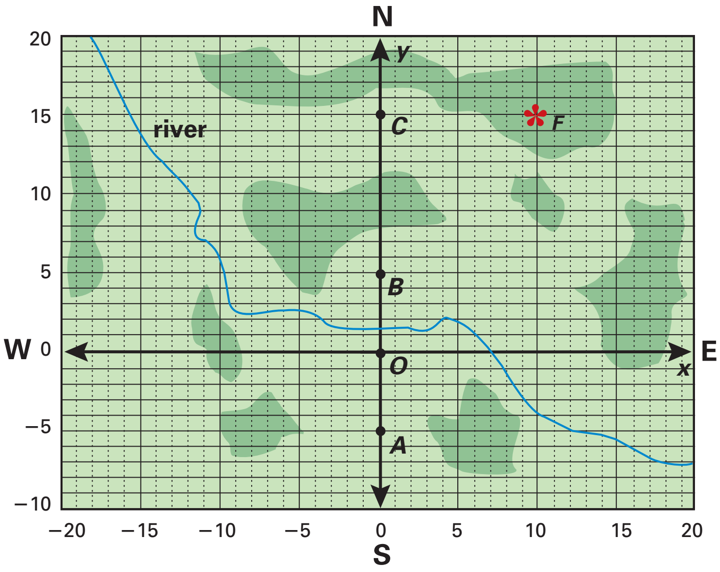
8. The coordinates of a point are both negative. In which quadrant does the point lie?

Use the map on page 3 to answer problems 9 and 10.

9. Find the point (−20, −5) on the computer screen on page 3. What can you say about the position of this point in relation to point *A*?

There is a fire at point *F* (10, 15).

10. What directions, measured in degrees, should be given to the firefighters at towers *A*, *B*, and *C*?



The computer screen can be refined with horizontal and vertical lines that represent a grid of distances 1 km apart. The side of each small square represents 1 km.

The screen on the left shows a river going from NW to SE.

11. **a.** What are the coordinates of the two points where the river leaves the screen?

b. What are the coordinates of the points where the river crosses the x-axis? Where does it cross the y-axis?

A fire is moving from north to south along a vertical line on the screen. The fire started at F (10, 15).

12. **a.** What are its positions (coordinates) after the fire has moved 1 km south? After it has moved 2 km south? After 3 km south? After 10 more kilometers south?

b. Describe what happens to the x-coordinate of the moving fire.

Vertical and horizontal lines have special descriptions. For example, a vertical line that is 10 km east of the origin can be described by $x = 10$.

13. **a.** Why does $x = 10$ describe a vertical line 10 km east of the origin?

b. How would you describe a horizontal line that is 5 km north of point O? Explain your answer.

14. **a.** Where on the screen is the line described by $x = -5$?

b. Where on the screen is the line described by $y = 15$?

c. Describe the path of a fire that is moving on the line $y = 8$.

The description $x = 10$ is called an **equation of the vertical line** that is 10 km east of O. An **equation of the horizontal line** that is 10 km north of O is $y = 10$.

Fire Regions

To prevent forest fires from spreading, parks and forests usually contain a network of wide strips of land that have only low grasses or clover, called *firebreaks*. These firebreaks are maintained by mowing or grazing.

In the forest, some firebreaks follow parts of the lines described by the equations $x = 14$, $x = 16$, $x = 18$, $y = 8$, $y = 6$, $y = 4$, $y = 2$, and $y = 0$.

15. a. Using **Student Activity Sheet 3**, draw the firebreaks through the wooded regions of the park.

b. Write down the coordinates of 5 points that lie north of the firebreak described by $y = 8$.

The fire rangers describe the region north of the firebreak at $y = 8$ with "y is greater than 8." This can be written as the *inequality* $y > 8$.

16. a. Explain how $y > 8$ describes the whole region north of $y = 8$.

b. Why is it not necessary to write an inequality for x to describe the region north of $y = 8$?

c. Describe the region west of the firebreak at $x = 14$ by using an inequality.

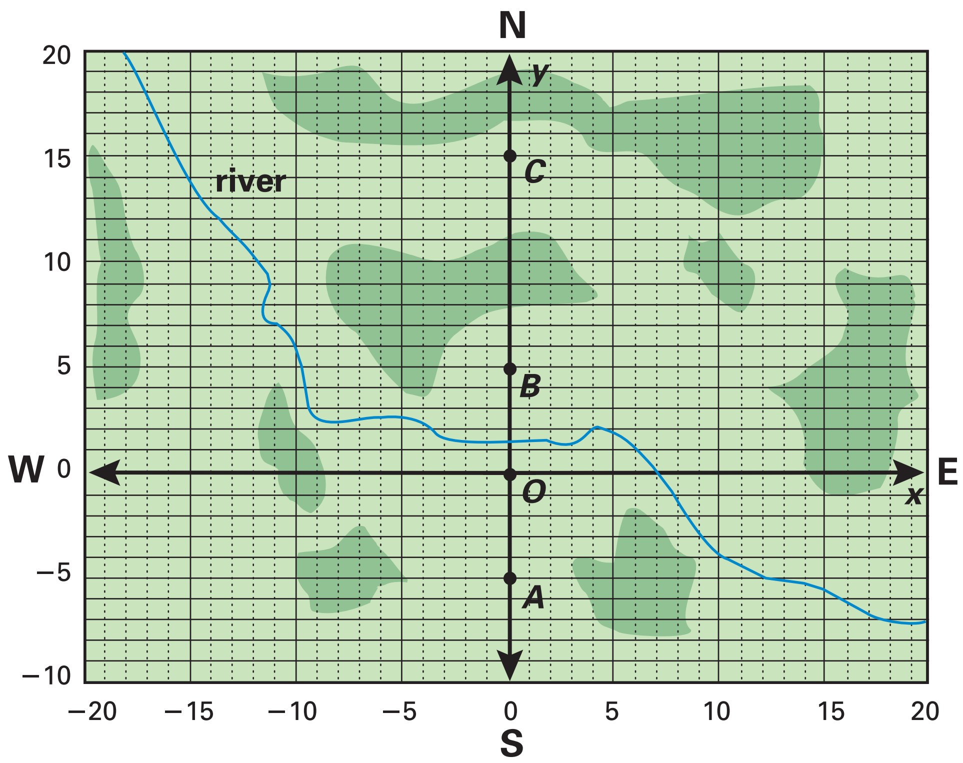
A fire is restricted by the four firebreaks that surround it. If a fire starts at the point (17, 5), then the vertical firebreaks at $x = 16$ and $x = 18$ and the horizontal firebreaks at $y = 4$ and $y = 6$ will keep the fire from spreading. Here is one way to describe the region:

x is between 16 and 18; *y* is between 4 and 6.

You can use inequalities to describe the region:

$16 < x < 18$ and $4 < y < 6$

This can also be read "*x* is greater than 16 and less than 18, and *y* is greater than 4 and less than 6."



Use **Student Activity Sheet 3** for problems 17 through 19.

17. Show the restricted region for a fire that starts at the point (17, 5).

18. Another fire starts at the point (15, 3). The fire is restricted to a region by four firebreaks. Show the region and use inequalities to describe it.

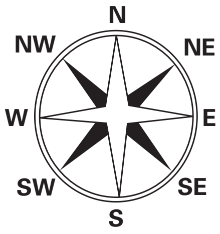
19. Use a pencil of a different color to show the region described by the inequalities $-6 < x < -3$ and $6 < y < 10$.

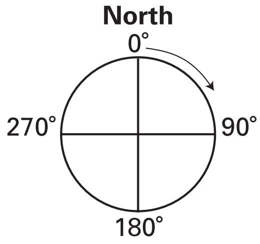
A Where There's Smoke

Summary

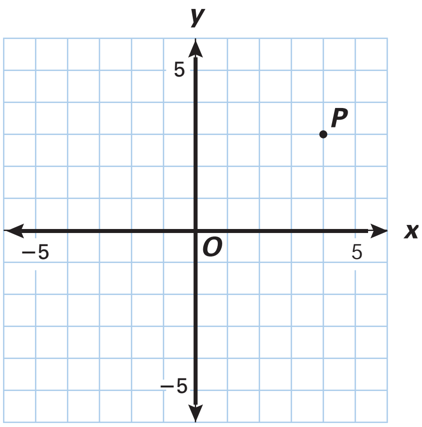
You have seen two ways to indicate a direction starting from a point on a map.

- Using a compass rose, you can indicate one of the eight directions: N, NE, E, SE, S, SW, W, and NW.
- You can indicate direction using degree measurements, beginning with 0° for north and measuring clockwise up to 360°.





Another way to describe locations on a map is by using a grid or coordinate system. In a coordinate system, the horizontal axis is called the *x-axis* and the vertical axis is called the *y-axis*. The axes intersect at the point (0, 0), called the *origin*.



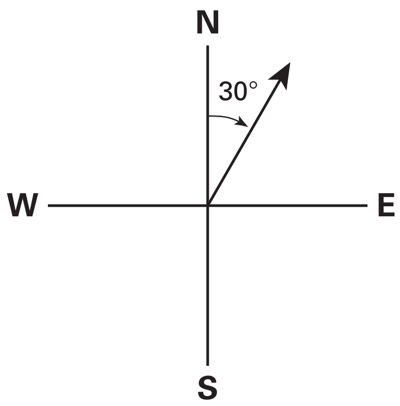
The location of a point is given by the *x*- and *y*-coordinates and written as (*x*, *y*).

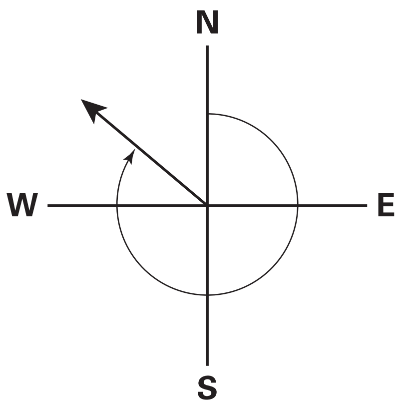
When points are on a vertical line, the *x*-coordinate does not change. Vertical lines can be described by equations such as $x = 1$, $x = 8$, and $x = -3$.

When points are on a horizontal line, the *y*-coordinate does not change. Horizontal lines can be described by equations such as $y = -5$, $y = 0$, and $y = 3$.

Inequalities can be used to describe a region. For example, $1 < x < 3$ and $-2 < y < 3$ describes a 2-by-5 rectangular region.

Check Your Work





1. a. The direction 30° is shown in the diagram above on the left. What direction is opposite 30°?

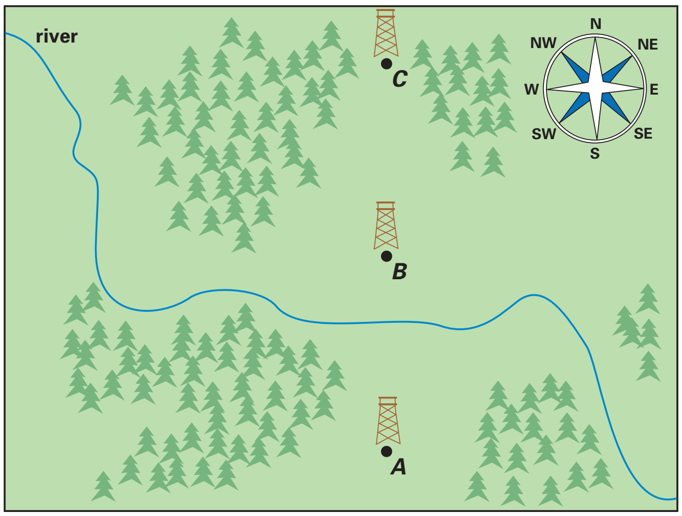
   b. What direction is shown above on the right? What degree measurement is the opposite of that direction?

A fire starts at point *F* (10, 15). A strong wind from the NE blows the fire to point *G*, which is 5 km west and 5 km south of *F*.

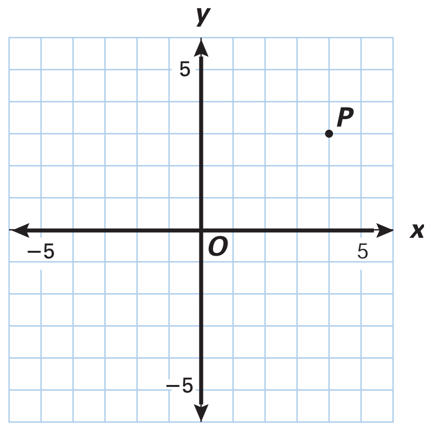
Note: You can the use the map on page 5 to see the situation.

2. a. What are the coordinates of *G*?

   b. What directions in degrees will fire towers *A* at (0, −5) and *C* at (0, 15) send to the firefighters?



3. One day, rangers report smoke at a direction of 240° from tower *A* and 240° from tower *B*. Is it possible that both reports for the same fire are correct? Why or why not?



4. a. Suppose point *P* in the coordinate system on the left moves on a straight line in a horizontal direction. What is an equation for that line?

b. Use an inequality to describe the region below the line.

5. In the coordinate system above, point *O* is the center of a rectangular region, and *P* is one corner. The boundaries of the region are horizontal and vertical lines. Use inequalities to describe the region.

For Further Reflection

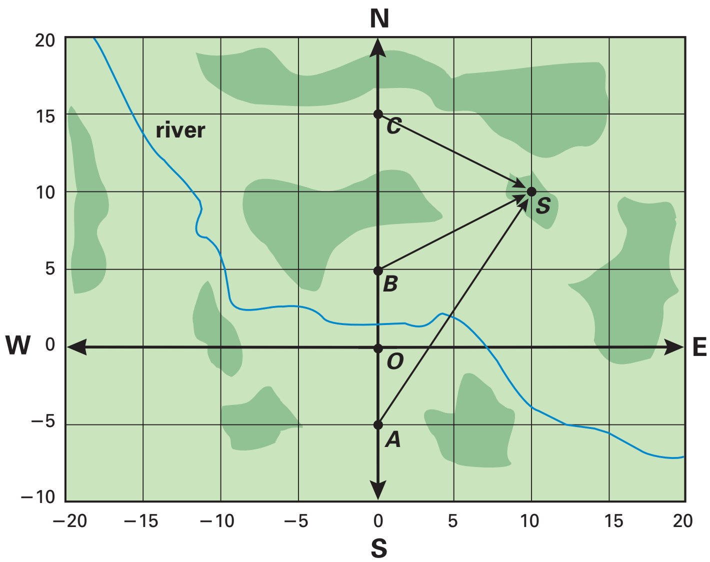
Compare the two ways to indicate a direction starting from a point on a map. Give one advantage of each.

B Directions as Pairs of Numbers

Directing Firefighters

In the previous section, directions from a point were indicated by compass references, such as N or NW. A second way to indicate directions involved using degrees measured clockwise from north, such as 30° or 210°. This section introduces a third method to indicate directions.

Smoke is reported at point *S* (10, 10). A firefighting crew is at tower *B*, so the crew needs to go 10 km east and 5 km north. Those instructions can be sent as the direction pair [+10, +5]. The first number gives the horizontal component of the direction, and the second number gives the vertical component.



B Directions as Pairs of Numbers

Note that direction pairs are in brackets like this: [,]. Coordinates of a point are in parentheses like this: (,).

1. **a.** Write a direction pair to describe the direction of the fire at point *S* as seen from point *A*.

b. Do the same to describe point *S* as seen from point *C*.

2. Using the top half of **Student Activity Sheet 4**, locate and label point *G* at (20,15). Then use direction pairs to describe the location of *G* as seen from points *A*, *B*, and *C*.

Notice that for the rangers at tower *B*, the direction to point *S* is the same as the direction to point *G*. So we can say that the direction pairs [+10, +5] and [+20, +10] indicate the same direction from point *B*.

3. **a.** Why are they the same?

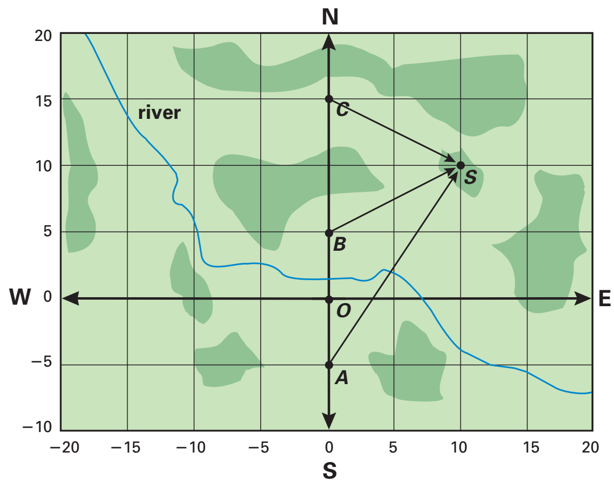
b. Write three other direction pairs that indicate this same direction from point *B*.

4. Find three different points on the map that are in the same direction from tower *A* as point *S*. Write down the coordinates of these points.

5. **a.** Give two direction pairs that indicate the direction NW.

b. Give two direction pairs that indicate the direction SE.

6. What compass direction is indicated by [+1, 0]? What compass direction is indicated by [0, −1]?



Use the graph on the top half of **Student Activity Sheet 4** for problems 7 through 9.

7. Locate the fire based on the following reports.

- Rangers at tower *B* observe smoke in the direction [−9, +2].
- Rangers at tower *C* observe smoke in the direction [−3, −1].

8. Do the direction pairs [−6, +9] and [−8, +12] indicate the same direction? Use a drawing as part of your answer.

9. **a.** Locate and label four points that are in the direction [+1, +1.5] from point *A*.

b. What is a quick way to draw all the points that are in the direction [+1, +1.5] from *A*?

10. For each two direction pairs below, explain why they indicate the same direction or different directions.

a. [+1, +3] and [+4, +12]

b. [−4, +3] and [+8, −6]

c. [+5, +8] and [+6, +9]

You can use many direction pairs to indicate a particular direction.

11. **a.** Give five direction pairs that indicate the direction [+12, +15].

b. What do all your answers to part **a** have in common?

c. Could any of the direction pairs you listed have fractions as components? Why or why not?

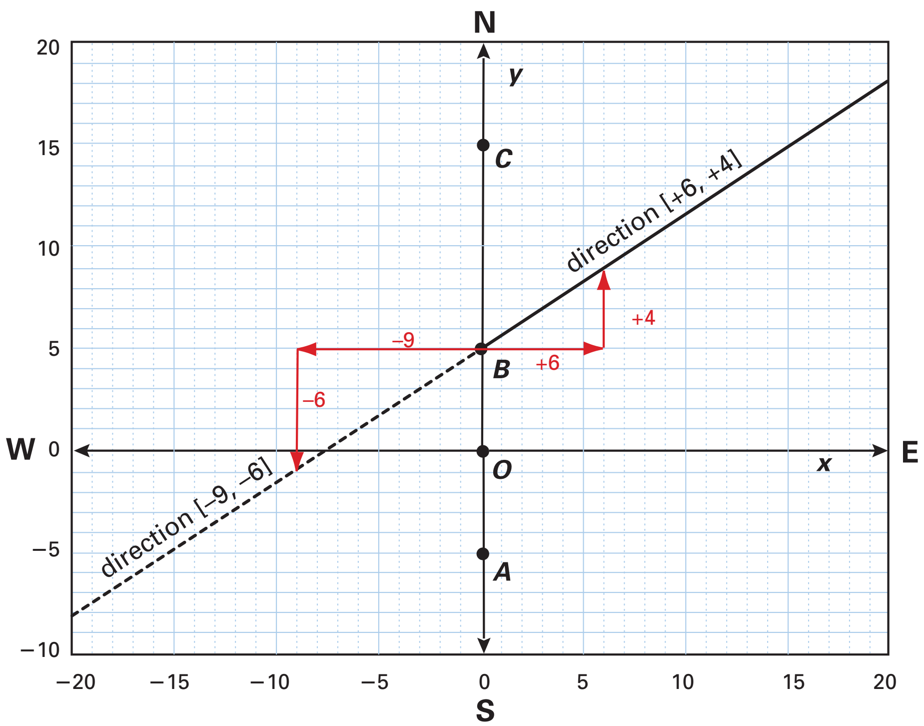
12. Use the map on the bottom of **Student Activity Sheet 4**.

a. Label the point *A* (0, −5) on the map.

b. Show all the points on the map that are in the direction [–1, +2] from *A.*

c. Show all the points on the map that are in the direction [+1, −2] from *A.*

d. What do you notice in your answers for parts **b** and **c**?



The two number pairs [+6, +4] and [–9, −6] represent opposite directions. All the points from B in the directions [+6, +4] and [–9, −6] are drawn in the diagram. The result is a line.

13. a. Give three other direction pairs on the solid part of the line through *B*.

b. Give three other direction pairs on the dotted part of the line through *B*.

c. What do all six direction pairs have in common?

14. Suppose you want to graph the line that has direction pair [+75, +25] and that starts at (0, 10). Describe how you might do this.

Up and Down the Slope

All the number pairs for a single direction and for the opposite of that direction have something in common: they all have the same ratio.

You can calculate two different ratios for a number pair:

horizontal component divided by vertical component

or

vertical component divided by horizontal component

Mathematicians frequently use this ratio: $\frac{\textbf{vertical component}}{\textbf{horizontal component}}$

and call that ratio the **slope** of a line. **slope** = $\frac{\textbf{vertical component}}{\textbf{horizontal component}}$

15. a. Find the slope of the line you drew in problem 12, using the direction [−1, +2] given in 12b.

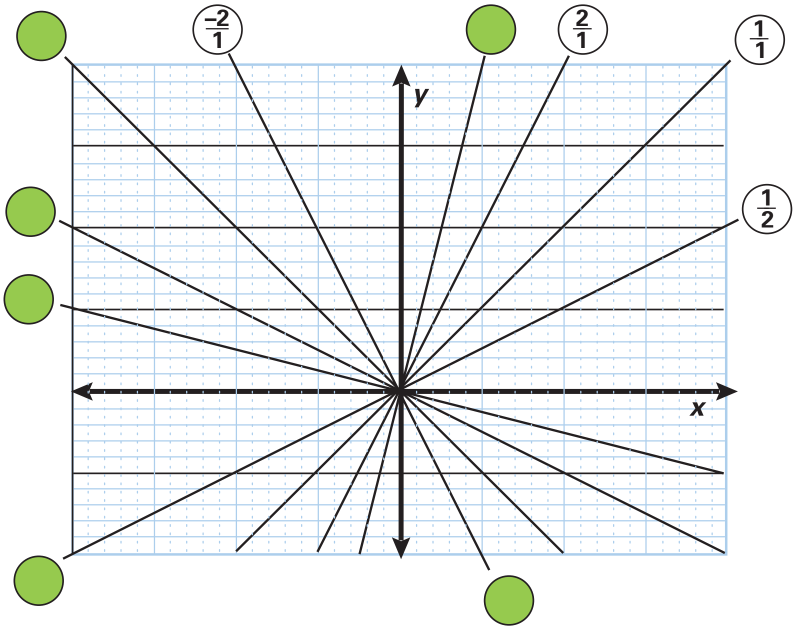
b. Do the same as in part **a,** but now use the direction [+1, −2] from 12c.

c. Reflect What do you notice if you compare your answers to problems 15a and 15b?

From problem 13, you can conclude that $\frac{4}{6} = \frac{-6}{-9}$.

16. a. Explain how you can conclude this from problem 13.

b. Using direction pairs, explain that $\frac{-4}{2} = -2$.



Use **Student Activity Sheet 5** for problems 17 through 19.

Each of the lines drawn on the **coordinate grid** contains the point (0, 0). For some of the lines, the slope is labeled inside its corresponding circle.

17. a Fill in the empty circles with the correct slope.

b. What is the slope for a line that goes through the points (1, 1) and (15, 3)? How did you find out?

18. a. What do you know about two lines that have the same slope?

b. Explain that $\frac{3}{1}$, $\frac{6}{2}$, $\frac{-3}{-1}$, and $\frac{15}{5}$ all indicate the same slope. What is the simplest way to write this slope?

19. Draw and label the line through (0, 0) whose slope is:

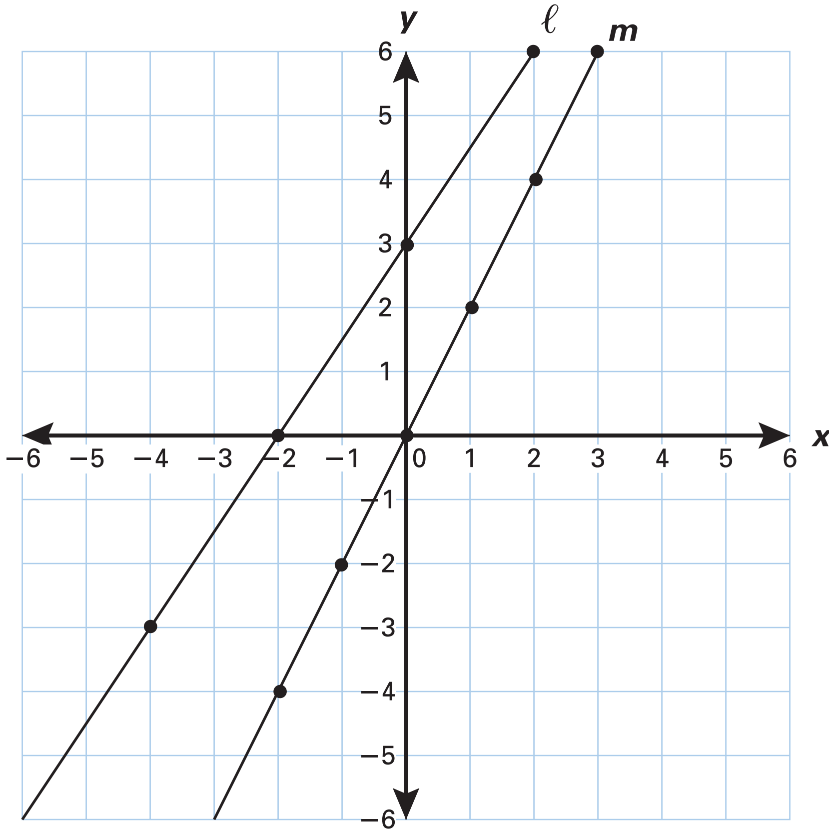
a. $\frac{4}{3}$

b. $-\frac{1}{2}$

The two lines in the graph below are not parallel.

20. **a.** Find the slope of each line.

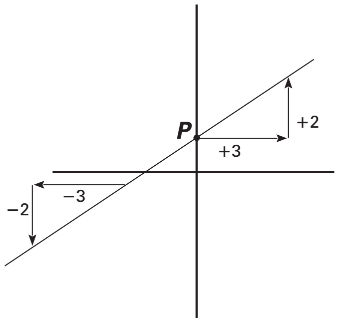
b. This grid is too small to show the point where the two lines meet. Find the coordinates of this point and explain your method for finding it.



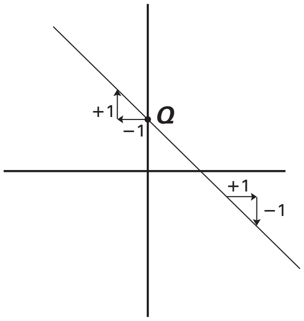
B Directions as Pairs of Numbers

Summary

You can indicate a direction from a point, using a direction pair such as [+3, +2] or [+1, −1]. The first number is the horizontal component, and the second number is the vertical component.



From *P*, the points in the directions [+3, +2] and [−3, −2] are on the same line. The slope of this line is $\frac{2}{3}$.



From *Q*, the points in the directions [+1, −1] and [−1, +1] are on the same line. The slope of this line is $\frac{+1}{-1} = -1$.

Brackets are used to distinguish direction pairs from coordinate pairs.

[+2, −4] is a direction pair.

(2, −4) are the coordinates of a point.

All direction pairs in the same and opposite direction have the same ratio.

The slope of a line is given by this ratio:

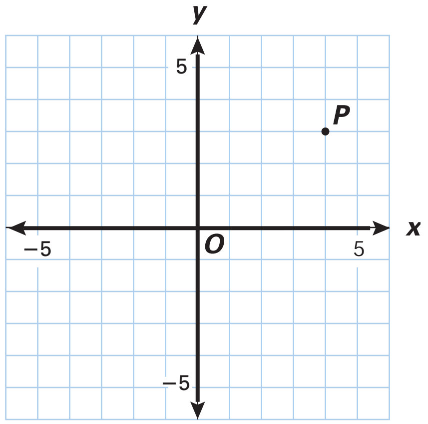
$$\textbf{slope} = \frac{\textbf{vertical component}}{\textbf{horizontal component}}$$

If you want to draw a line whose slope is given, you may want to find a direction pair first that fits the given slope.

Check Your Work

1. a. Give the coordinates of point *P* in this coordinate system.

   b. Give two direction pairs that describe the direction from *O* to point *P* in the coordinate system.



   c. Copy the drawing in your notebook. Locate and label three points that are in the direction [−4, −2] from point *P*.

   d. What is a quick way to draw all points in the direction [−4, −2] from point *P*?

2. For each two direction pairs below, say whether they indicate the same or different directions and explain why.

   a. [+4, +3] and [+8, −6]

   b. [+5, +8] and [+1, +1.6]

   c. [+13, 0] and [+25, 0]

   d. [+0.5, +2] and [+2, +8]

3. a. Draw a coordinate system in your notebook like the one for problem 1; mark point *P* from problem 1 in the grid you drew. Mark point *Q* with coordinates (1, 1).

   b. What direction pair describes the direction from *P* to *Q*?

   c. Draw the line through *P* and *Q* and find its slope.

B Directions as Pairs of Numbers

4. In the coordinate system you drew for problem 3, draw and label the line *m* through *O* (0, 0) that has a slope of 2.

5. **a.** How many lines contain both points (1, 2) and (26, 52)? Explain your reasoning.

b. Find the slope of the line(s) in part **a.** How did you find it?

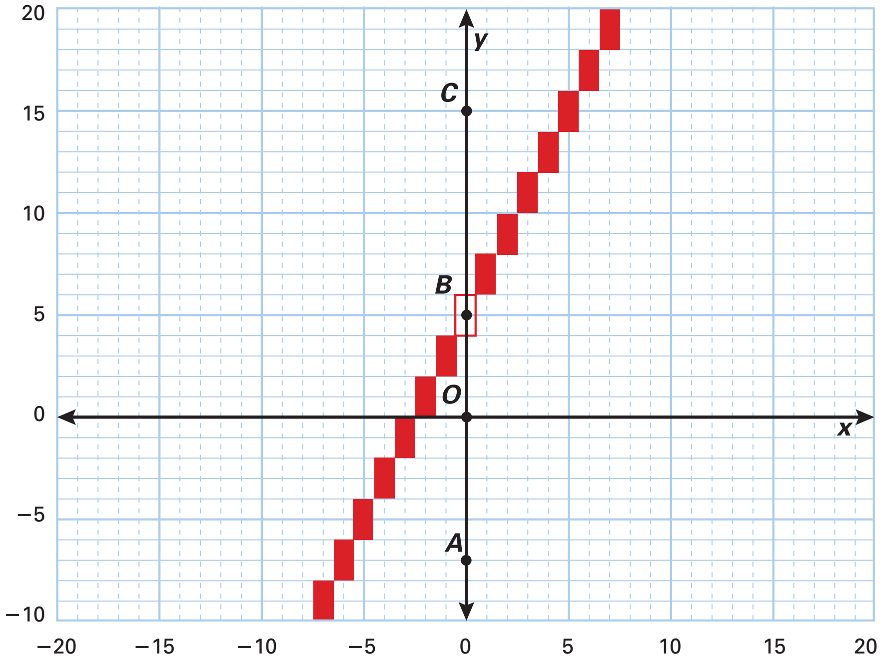
For Further Reflection

How can similar triangles be used to find the slope of a line?

C An Equation of a Line

Directions and Steps

In the coordinate system below, a line is drawn in the direction [+1, +2] from *B*.



You can think of moving along this line one step at a time. Each step is a move of +1 unit horizontally and +2 units vertically.

(0, 5)
+1 ⇩ ⇩ +2
(1, 7)
+1 ⇩ ⇩ +2
(2, 9) etc.

1. a. The description shows two steps along the line. Where are you after 10 steps?

b. Where are you after 25 steps? After 1,000 steps?

This description shows steps along the same line but in the opposite direction.

(0, 5)
−1 ⇩ ⇩ −2
(−1, 3)
−1 ⇩ ⇩ −2
(−2, 1) etc.

2. a. Where are you after 10 steps?

b. After 100 steps?

C An Equation of a Line

A computer or graphing calculator can quickly calculate and draw all of the points on a line. Suppose a computer takes horizontal steps of +0.1 and −0.1 when drawing the points on this line.

3. a. What are the corresponding vertical distances for each step the computer takes?

b. If you start at (0, 5), where are you after 8 steps when +0.1 is the horizontal distance?

c. If you start at (0, 5), where are you after 3 steps when −0.1 is the horizontal distance?

Here is a rule you may have discovered.

Starting point: (0, 5).
After 100 horizontal steps of +1:
$x = 100$
$y = 5 + 100 \times 2 = 205$

4. a. Explain what each of the numbers in $y = 5 + 100 \times 2 = 205$ refers to.

b. Write a similar rule for 75 horizontal steps of +1.

c. Write a rule for 175 horizontal steps of +1.

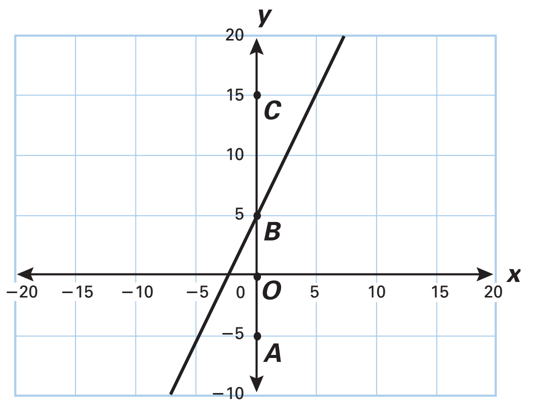
d. Write a rule for $3\frac{1}{2}$ horizontal steps of +1.

From the rules you wrote in problem 4, you can find a formula relating the *x*-coordinates and the *y*-coordinates:

$$y = 5 + x \cdot 2 \qquad \text{or} \qquad y = 5 + 2x$$

5. a. Explain the formula.

b. Does the formula work for negative values of *x*?



The formula $y = 5 + 2x$ is called an **equation of a line**. If you draw a graph for this equation, you see a line like this.

In the equation $y = 5 + 2x$, two numbers play special roles.

6. a. What is the importance of the "5" for the graph?

b. What is the importance of the "2" for the graph?

There are special names for the 5 and the 2 in the equation $y = 5 + 2x$. The 2 is called the *slope*, and the 5 is called the *y-intercept*.

7. Why do you think it is called the *y*-intercept?

8. Using the graph on page 22 write the equation for a line that goes through point *C* and has a slope of 2.

9. Make a copy of the graph shown on page 22 on a piece of graph paper.

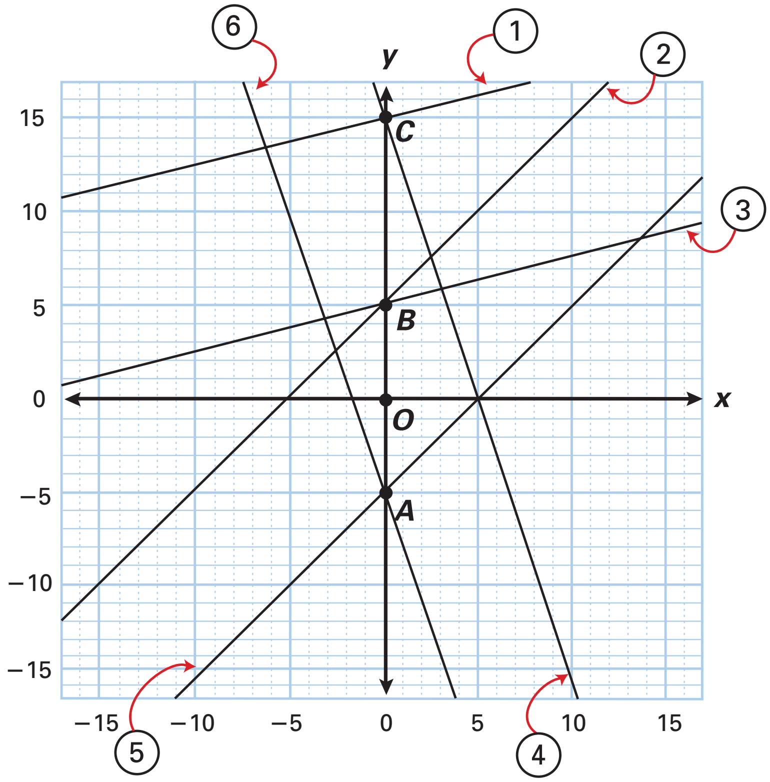
a. Show the line through *B* with slope $\frac{1}{2}$. Then label the line with its equation.

b. Show the line through *C* with slope $\frac{3}{6}$ and label the line with its equation.

c. What do you notice about the two lines? Justify your answer.

These two equations represent the same line:

$$y = 5 + (-2) \cdot x \qquad \text{and} \qquad y = 5 - 2x$$



10. Explain why the equations represent the line through *B* with slope −2.

11. a. Write an equation for the line that contains *B* and forms a 45° angle with the direction east.

b. What is the equation if the line contains *O* instead of *B*?

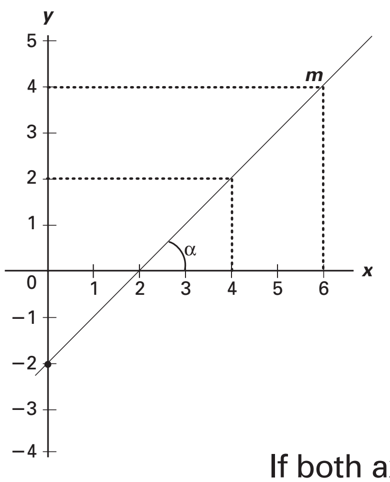
12. a. In your notebook, write the equation for each of the six lines in the grid to the left.

b. Which lines are parallel? Explain your answers.

c. For all equations, find the value of *y* for $x = 0$. What do you notice?

What's the Angle?

Line m is drawn in the coordinate system below.



13. a. What is the slope of line m? What is its y-intercept?

b. Write an equation for line m.

c. Measure angle α (the Greek letter alpha).

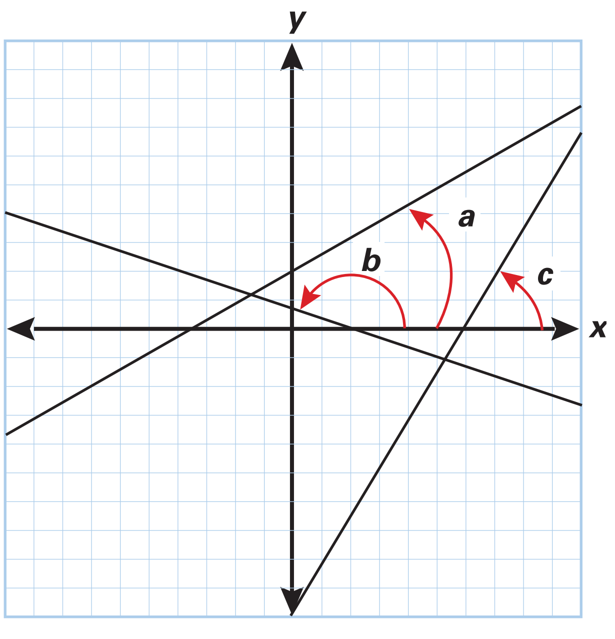
14. a. On graph paper, draw two lines in a coordinate system like the one here—one that forms a 30° angle with the x-axis and one that forms a 60° angle with the x-axis.

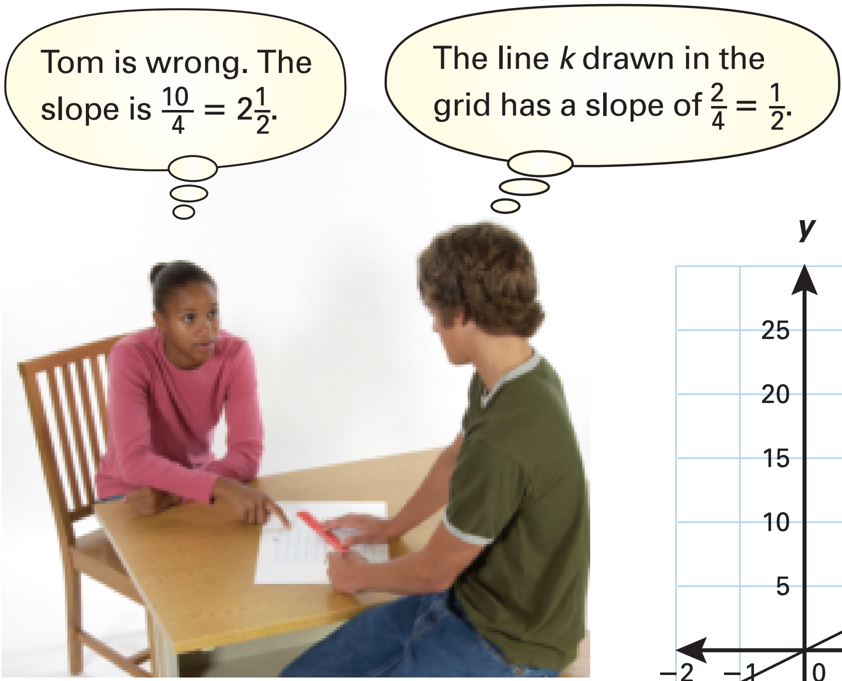
b. Estimate the slope of each line.

If both axes are scaled in the same way, there is an angle that corresponds to every slope. The slope is then equal to the **tangent ratio** for that angle, abbreviated *tan*.

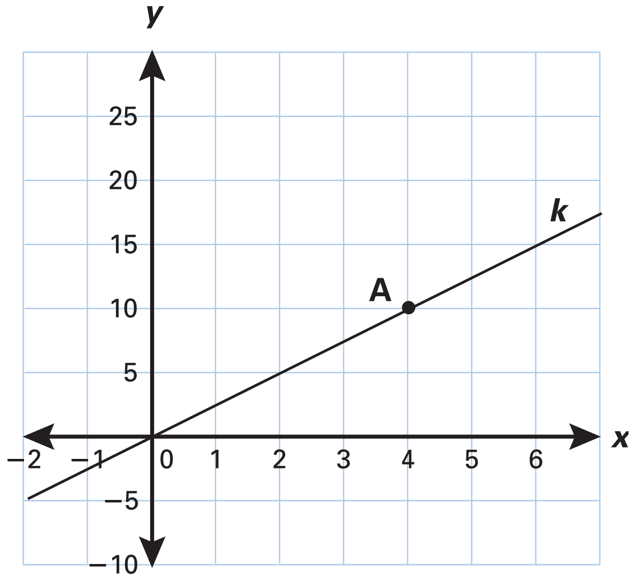
$$\textbf{slope} = \textbf{tan}\ \alpha = \frac{\textbf{vertical component}}{\textbf{horizontal component}}$$

15. Find the slope and measure the angle for each of the lines in the grid below. Note: The axes are scaled in the same way.





A grid does not always have the same scaling on both axes. If the scales are different, you cannot use the tan of the angle as slope.



16. **a.** Do you agree with Tom or Brenda? Explain your answer.

b. In your notebook, copy the grid above and draw a line through (0, 20) with slope −1.

c. Write the equation of the line you drew in part **b**.

17. **a.** Measure the angle that line *k* makes with the *x*-axis.

b. Draw a grid with equal scaling on both axes and draw a line *k*, through O (0, 0) and A (4, 10), in the grid.

c. Measure the angle that line *k* makes with the positive *x*-axis in the grid you drew for part **b**.

d. Which of the two angles you measured for line *k*, the one in part **a**, or the one in part **c**, corresponds to the slope? Give reasons for your answer.

18. **a.** What can you say about a line and its slope if the angle is 0°?

b. Will the angle that a line makes with the positive *x*-axis be greater than 90°? Explain your thinking.

19. If a line goes through (2, 3) and has slope 4, how could you find the *y*-intercept?

C An Equation of a Line

Summary

All straight lines can be determined by a point and a direction. The direction is called *slope*. An equation for the line that contains the point (0, 5) and has slope 3 is $y = 5 + 3x$.

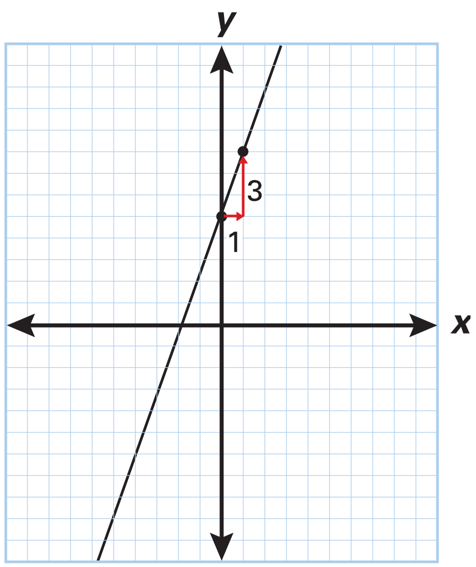
The number 5 indicates the **intercept** on the y-axis, which is a very special point.

The number 3 is the value of the slope.

The equation of a line that is not vertical has this form:

y = intercept + slope • x

If you have the equation of a line, you can find the y-intercept by calculating the y-value for $x = 0$. The y-intercept of a line may be positive, zero, or negative. The slope of a line may be positive, zero, or negative.



Another way to describe the slope is using the tangent of the angle the line makes with the x-axis. You have to be careful with slope and tangents when the two axes in a grid are not scaled in the same way.

Check Your Work

1. a. What is the y-intercept of the line with equation $y = -3 + 2x$? What is its slope?

   b. Write the equation for a line that goes through (0, 0) and has the same slope as the equation in part **a**.

2. a. Write an equation for the line through C (0, 15) with slope $-\frac{1}{4}$.

   b. Write an equation for the line through A (0, −5) with slope −1.

3. Draw a coordinate system in your notebook. Draw the lines defined in problem 2 in this coordinate system.

4. a. Write an equation of a line with a positive y-intercept and a negative slope. Draw this line in a coordinate system.

   b. What can you tell about a line with a y-intercept equal to 0?

   c. What can you tell about a line whose slope equals 0?

For Further Reflection

Describe in your own words what is meant by the word *slope*. In your description, also explain why it is important to be careful when finding the slope if the scaling on the x-axis is different from the scaling on the y-axis. You may use one or more examples in your description.

D Solving Equations

Jumping to Conclusions

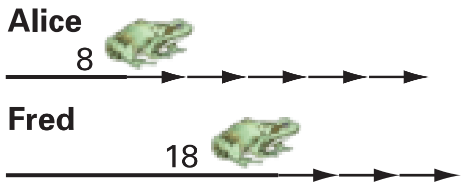
The activities in the previous sections involved coordinates and directions. The activities led to investigating the slope and equation of a line. This section takes a look at writing and solving equations.

Two frogs, Alice and Fred, are near a path in a forest. Suddenly they hear footsteps on the path. To avoid possible danger, they jump away from the path.

Alice begins 8 decimeters (dm) from the path, and Fred begins 18 dm from the path. (Note: 10 decimeters = 1 meter.) Each frog takes several jumps and then stops.

1. What information would you need to find their new distances from the path?

Suppose that Alice and Fred travel the same distance with each jump, but Alice takes 5 jumps and Fred takes 3 jumps. The diagram below illustrates their new positions.



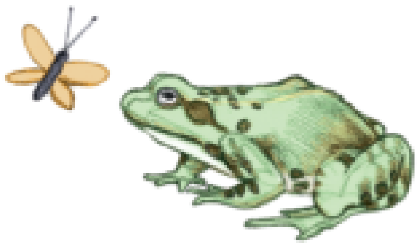
Suppose Alice and Fred travel 4 dm with each jump.

2. a. Find the distance from the path to each frog's new position. Draw a diagram showing this situation.

   b. Suppose you know that each jump is between 2 dm and 6 dm. What can you conclude about where each frog finishes?

Suppose the frogs finish their jumps at exactly the same distance from the path, and you want to know the distance of each jump and each frog's final distance from the path.

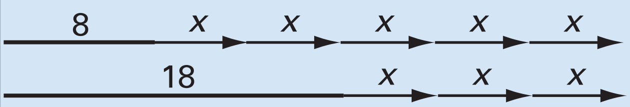
3. Write down your thinking about this problem. Share your group's method with the other members of your class.



One way to answer problem 3 is to label the missing value, or the **unknown**. In this problem, the unknown is the length of each jump. You can use the symbol x for the length of a jump. The box below gives a diagram and an equation for answering problem 3.

Box A

4. Explain how the equation $8 + 5x = 18 + 3x$ describes the diagram in Box A.

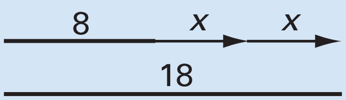


$$8 + 5x = 18 + 3x$$

Study the diagrams and equations to see the steps in finding the length of a jump.

Box B

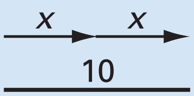
5. Explain the equation in Box B and describe how the diagram was changed from Box A to Box B.



$$8 + 2x = 18$$

Box C

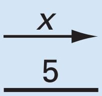
6. Explain the equation in Box C and describe how the diagram was changed from Box B to Box C.



$$2x = 10$$

Box D

7. Explain the equation in Box D and describe how the diagram was changed from Box C to Box D.



$$x = \frac{10}{2} = 5$$

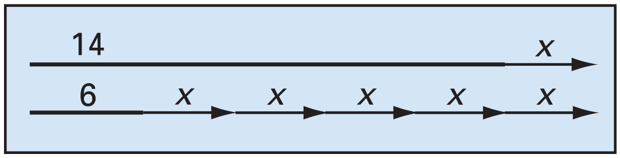
8. Write a “frog problem” and an expression to represent each diagram in parts **a** and **b**.

a. 

b. 

9. a. Write a “frog problem” and an equation to represent the diagram in Box A below.

Box A



b. Draw the diagram for the next step in finding the value of *x* and write the equation for your diagram.

c. Complete the sequence of diagrams and equations.

10. Here is an equation: $12 + 2x = 6 + 4x$.

a. Use a sequence of boxes to solve the equation. Start by drawing a diagram to represent each side of the equation.

b. Draw the rest of the boxes and diagrams to solve the equation.

11. a. Describe the equation $11 + 9x = 26 + 4x$ as a “frog problem.”

b. Find the value of *x* in the equation and explain the steps in your solution. You may want to use a series of boxes, diagrams, and equations as part of your explanation.

12. Solve each equation for its unknown value and explain your method. How can you be sure that your answers are correct?

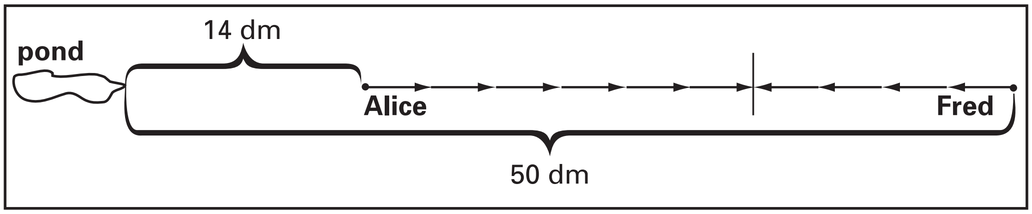
a. $100 + w + w = 75 + w + w + w + w$

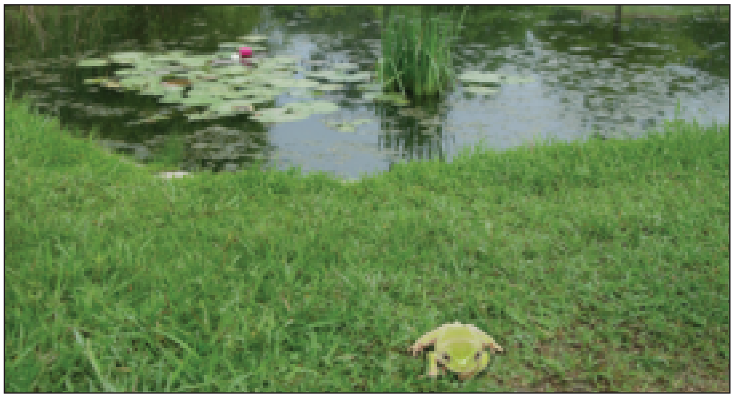
b. $y + 42 + y = 12 + 3y + 2y$

c. $144 + z = 120 + 9z$

Opposites Attract

One day, while exploring their territory, Alice is 14 dm from the pond and Fred is 50 dm from the pond. They start jumping toward each other. As shown below, they met after Fred took four jumps toward Alice and Alice took six jumps toward Fred.

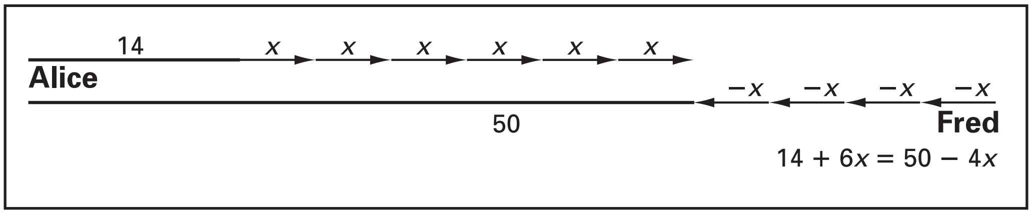




13. Suppose both frogs travel the same distance x with each jump.

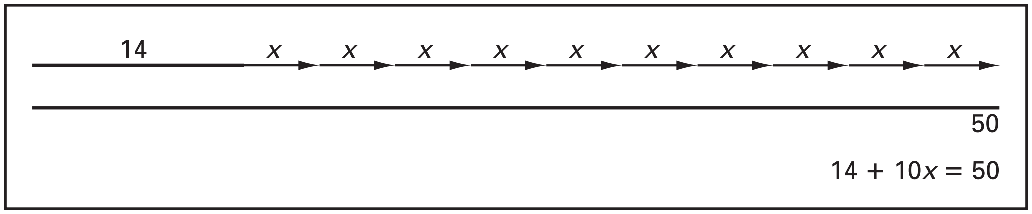
a. Explain how the diagram and equation in Box A represent the frogs' positions.

Box A



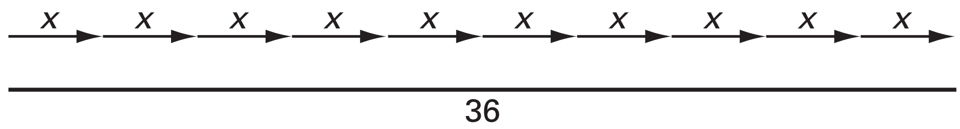
b. Explain the equation in Box B and describe how the diagram was changed from Box A to Box B.

Box B



14. **a.** Explain the equation in Box C and describe how the diagram was changed from Box B to Box C.

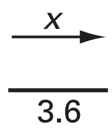
Box C



$10x = 36$

b. Explain the equation in Box D and describe how the diagram was changed from Box C to Box D.

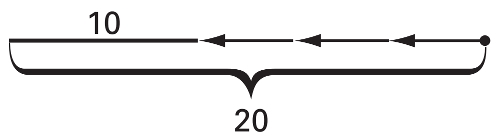
Box D



$x = \frac{36}{10} = 3.6$

15. Write a "frog problem" to represent Box A below. You can use Fred and Alice, or you may introduce new characters and situations. Be sure to solve your problem.

Box A



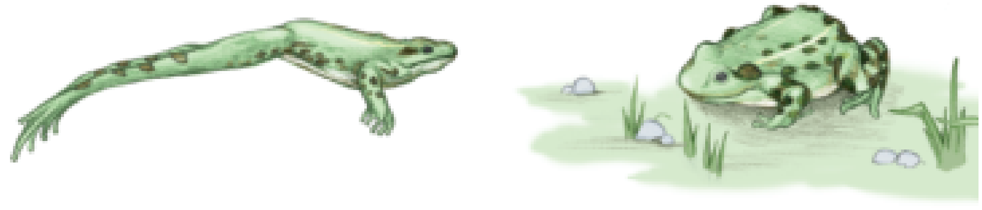
16. Draw a diagram to represent the expression $5 - 4x$.

17. **a.** If you start with the equation $27 - 5w = 7 + 3w$, explain why $27 = 7 + 8w$.

b. Solve the equation.

Number Lines

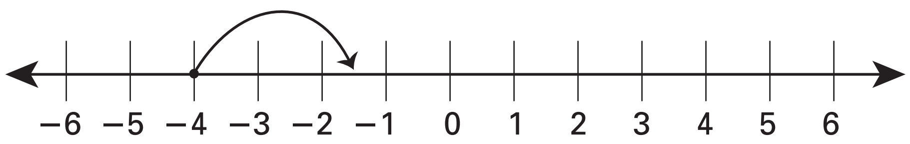
Frog problems can also be diagrammed on number lines.



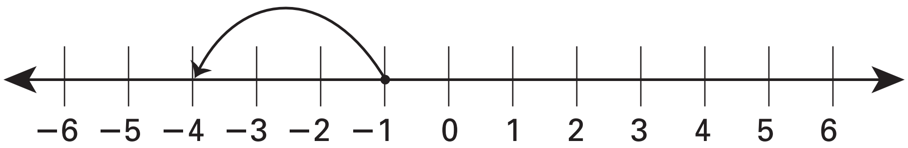
A number line has a positive and negative direction.

Jumps on the number line are considered positive if they move in a positive direction and negative if they move in a negative direction.

Here is an example.



Starting point is −4; jump is +2.5



Starting point is −1; jump is −3

Fred starts at the point −3 and makes 17 jumps in a positive direction. Alice starts at the point 2 and makes 12 jumps in a positive direction. They end at the same point. Assume that every jump is the same length, and use the letter k for that unknown length.

18. **a.** Write an equation for this situation.

b. Find the value of k.

c. Use a number line to check your solution.

19. The equation $8 + 12x = 3 + 2x$ represents a different "frog problem."

a. Use a number line to explain why the jumps must be in the negative direction.

b. Solve the equation.

In this section, you have solved equations using diagrams and a number line. Another method is to perform the same operation (addition, subtraction, multiplication, or division) on each side of an equation. Here is an example.

$15 + 8x = 37 - 3x$

Add $3x$ to both sides.

$15 + 11x = 37$

Subtract 15 from both sides.

$11x = 22$

Divide both sides by 11.

$x = 2$

20. Study the steps of the example shown above. Use similar steps to complete the solution for this equation.

$-5 - 6x = 1 - 9x$

Add $9x$ to both sides.

______ = ______

Add 5 to both sides.

______ = ______

____

______ = ______

21. Here are other steps to solve the same equation as in problem 20. Complete the solution and check to see if the result is equal to the result in problem 20.

$-5 - 6x = 1 - 9x$

Add $6x$ to both sides.

______ = ______

Subtract 1 from both sides.

______ = ______

____

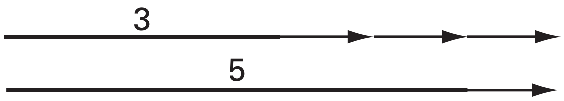
______ = ______

D Solving Equations

Summary

In this section, you used frog problems to write and solve equations. You solved equations by drawing diagrams, by using number lines, and by performing an operation (adding, subtracting, multiplying, or dividing) on each side of the equation.

For example, frog A starts 3 dm from a log, and frog B starts 5 dm from the same log. Frog A and B take jumps that are the same length. Frog A takes 3 jumps and frog B takes 1 jump and they meet in the same location, as shown in the diagram below.



To find out how long the jumps were, you solve the equation for the problem.

$$\begin{aligned} 3 + 3x &= 5 + x \\ 3 + 2x &= 5 \\ 2x &= 2 \\ x &= 1 \end{aligned}$$

Check Your Work

1. Write a "frog problem" and an expression to represent this diagram.

2. **a.** Draw a diagram representing this equation: $2 + 3x = 10 + x$.

   b. Solve the equation by changing the diagram step by step.

3. Here is an equation for a "frog problem" where the frogs are jumping in opposite directions and they meet.

   $$20 + 2v = 26 - 2v$$

   Solve the equation. Then check your answer using a number line.

4. Solve each equation. You may use any method from this section.

   a. $12 + u = 11 + 3u$

   b. $-4 + 2w = 2 + w$

   c. $10 - v = 24 + v$

5. Write three new "frog problems"— one that you think is easy, one that is more difficult, and one that is very difficult. Describe how to solve each problem.

For Further Reflection

Think about the three different methods for solving an equation. What are the advantages and disadvantages of each method?

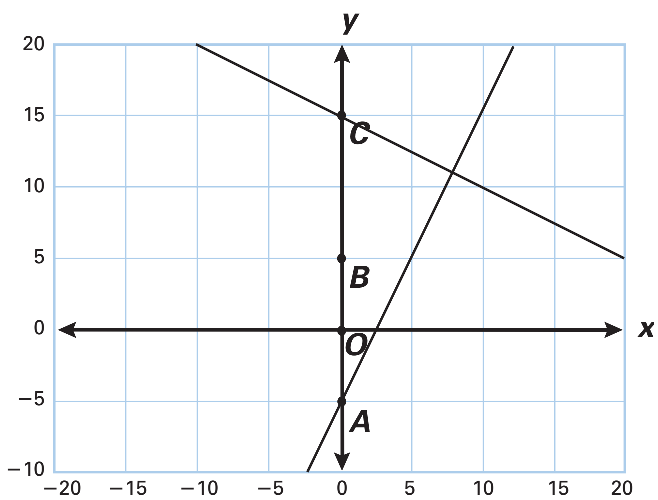
E Intersecting Lines

Meeting on Line

Let's return to the park rangers.

Rangers at tower *A* report a fire on the line whose equation is $y = -5 + 2x$.

Rangers at *C* report a fire on the line $y = 15 - \frac{1}{2}x$.



The two lines are displayed on a computer screen, as shown.

1. a. Explain how you can verify that the two lines on the screen represent the two equations.

   b. Using the screen, estimate the coordinates for the fire.

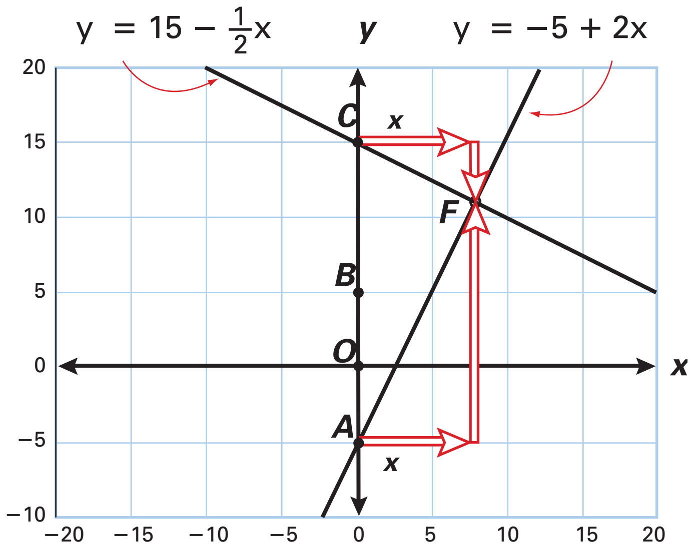
   c. How can you check your coordinates using both equations?

What's the Point?

Here is another way to find the coordinates of point *F*, the intersection of the two lines.

Think about the change from point *A* to point *F* as a horizontal step followed by a vertical step. Suppose the length of the horizontal step is represented by *x*.

2. a. Write an expression for the length of the vertical step.



The change from point *C* to point *F* is the same horizontal step x followed by a vertical step.

b. Write an expression for the length of that vertical step.

From the diagram above, you can set up the following equation:

$$-5 + 2x = 15 - \frac{1}{2}x$$

3. a. Write a "frog problem" to go with the equation.

b. Solve the equation using one of the methods from the previous section.

c. How can you use your answer from part **b** to find the *y*-coordinate of *F*?

The park supervisor has just received two messages:

Smoke is reported on the line $y = 15 - x$.

Smoke is reported on the line $y = 5 + 4x$.

4. **a.** Which tower sent each message?

b. Calculate the coordinates of the smoke.

5. Repeat problem 4 for these two messages:

Smoke is reported on the line $y = 5 + x$.

Smoke is reported on the line $y = -5 + 1\frac{1}{4}x$.

The park supervisor received the message $y = 15 + 2x$ from tower *C* and the message $y = 5 + 3x$ from tower *B*.

6. What message do you expect from tower *A*?

7. Make up your own set of messages and find the location they describe.

Use **Student Activity Sheet 6** for problems 8 and 9.

8. **a.** Draw the line $y = 5$; label it *l*. Draw the line $y = -3 + 2x$ and label it *m* in the coordinate system.

b. Find the point of intersection of the two lines on the graph; write down the coordinates.

c. Use the equations of the lines to check whether the coordinates you found in **b** are correct.

9. **a.** In the same coordinate system you used for problem 8, draw the line $y = 4 - 2x$; label it *n.*

b. Estimate the coordinates of the point of intersection of lines *m* and *n*.

c. Solve the equation $-3 + 2x = 4 - 2x$.

d. Are your answers for **b** and **c** the same? Explain why or why not.

Suppose the two lines $y = 10 + 2x$ and $y = -8 + 2x$ are on the park rangers' computer screen.

10. What can you tell about these lines? Do they have a point of intersection?

11. Look back at the graph for problem 20 on page 17.

a. Write an equation for each line in the graph.

b. Use the equations to find the coordinates of the point of intersection.

c. Compare the answer you found for part **b** to your answer from Section B.

Math History

Marjorie Lee Browne

Marjorie Lee Browne loved mathematics and studied the subject to the highest possible standards. She was one of the first African-American women in the United States to obtain a Ph.D.

She was born on September 9, 1914, in Memphis, Tennessee. Her father, a railway postal clerk, was very good at mental arithmetic, and he passed on his love of mathematics to his daughter. Her stepmother was a schoolteacher.

Browne taught at Wiley College, in Marshall, Texas, from 1942 to 1945. She received her Ph.D. from the University of Michigan in 1949. She taught mathematics at North Carolina Central University. For 25 years she was the only person in the department with a Ph.D.

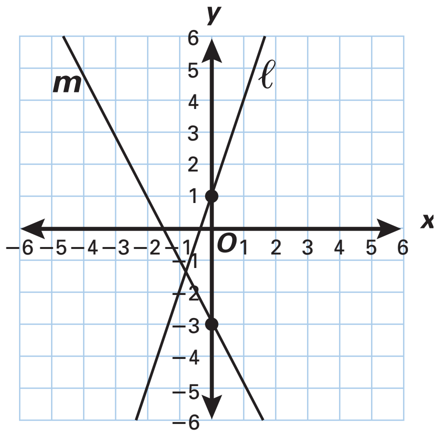
Browne used her own money to help gifted mathematics students continue their education. She will be remembered for helping students prepare for and complete their Ph.D.'s, encouraging them to do what she had achieved.

E Intersecting Lines

Summary

An equation for line ℓ is $y = 1 + 3x$, and the equation for line m is $y = -3 - 2x$.

You can try to find the point of intersection of lines l and m by reading the graph. Often this method will give you an estimate and not an accurate answer. Always use equations to check your result from reading the graph.



You can find the point where these two lines intersect by solving the following equation for x:
$-3 - 2x = 1 + 3x$

This will always give the exact result. You may use any method you used in Section D for solving the equation.

Check Your Work

1. What is the point of intersection of line $y = 3$ and line $x = -1$? How did you solve this problem?

2. a. Draw a coordinate system like the one on **Student Activity Sheet 6** and draw the line $y = -2 + 4x$.

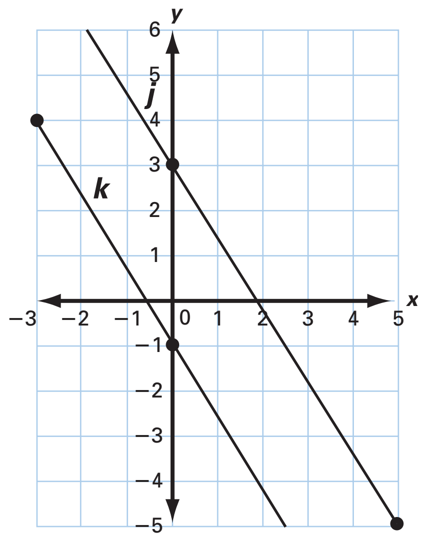
   b. Write an equation for a line that has no point of intersection with the line from part **a.**

   c. Draw line $y = 4 + x$ in the same coordinate system and find the point of intersection of the two lines you drew. Explain how you know your answer is correct.

3. a. Find the x-value of the intersection of the lines shown in the Summary by solving the equation $-3 - 2x = 1 + 3x$.

   b. Find the y-value of the point of intersection of lines m and ℓ shown on this graph.

4. Write an equation for each line shown in the graph. Then use the equations to find the intersection of the two lines.



For Further Reflection

Graphs and equations can be used to describe lines and their intersections. Tell which is easier for you to use and explain why.

Additional Practice

Section A Where There's Smoke

Here is a map of the San Francisco Bay Area. There are seven airports located in this area. People in the control tower at each airport can see the control towers at the other airports.

Source: © Rand McNally.

Use degree measurements with 0° for north and measure in a clockwise direction to answer the following questions.

1. a. In what direction from the San Carlos airport is the Hayward airport?

   b. Looking from Oakland in the direction 335°, you can see the Alameda airport. What is the opposite direction of 335°? Which airport is approximately in that direction?

   c. From the Hayward airport, you can see a tall skyscraper in the direction 300°. This same skyscraper can be seen from the San Francisco airport in the direction 350°. Describe the location of this skyscraper on the map.

A grid has been put over the map of the San Francisco Bay Area. The seven airports in this area are marked with planes. The San Francisco airport has the coordinates (0, 0).

Source: © Rand McNally.

2. **a.** What are the coordinates of the Oakland airport?

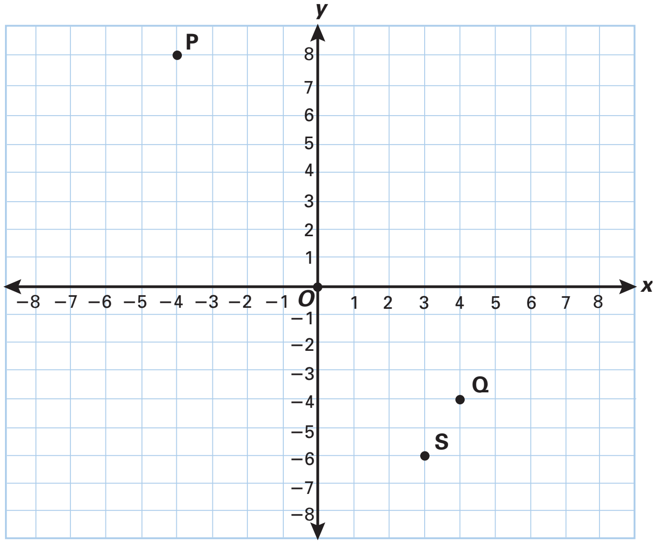
b. Sausalito Harbor is at coordinates (–4, 9). What is the equation of the line that is due north from Sausalito Harbor?

The San Mateo Bridge crosses the bay.

3. **a.** Use graph paper to draw the rectangular region that completely encloses the San Mateo Bridge. Use horizontal and vertical lines from the grid.

b. Use inequalities to describe the region you drew in part **a.**

Section B Directions as Pairs of Numbers



1. Describe point *P* on the graph as seen from the origin, using a pair of direction numbers.

2. Would point *Q* be on a line drawn through *O* and *P*? Explain why or why not.

3. Would point *S* be on a line drawn through *O* and *P*? Explain why or why not.

4. What is the slope of a line from point *Q* to point *P*?

Section C An Equation of a Line

1. **a.** On a sheet of graph paper, draw a line with a positive *y*-intercept and a negative slope. Call this line ℓ.

   b. What is the equation of line ℓ?

   c. What can you say about any line that is parallel to ℓ?

2. **a.** Draw a line with a negative *y*-intercept and a positive slope. Call this line *m*.

   b. Now draw a line that intersects line *m*. What is the slope of this line, and what is the intercept? What are the coordinates of the point of intersection of these two lines?

3. **a.** Draw a line whose equation is $y = 2x - 4$.

   b. What is the equation of the line that goes through (0, 0) and intersects the line $y = 2x - 4$ at the point (6, 8)?

Section D Solving Equations

1. Draw a diagram to illustrate each of the following equations. Then solve each equation.

   a. $12 + 2x = 5 + 4x$

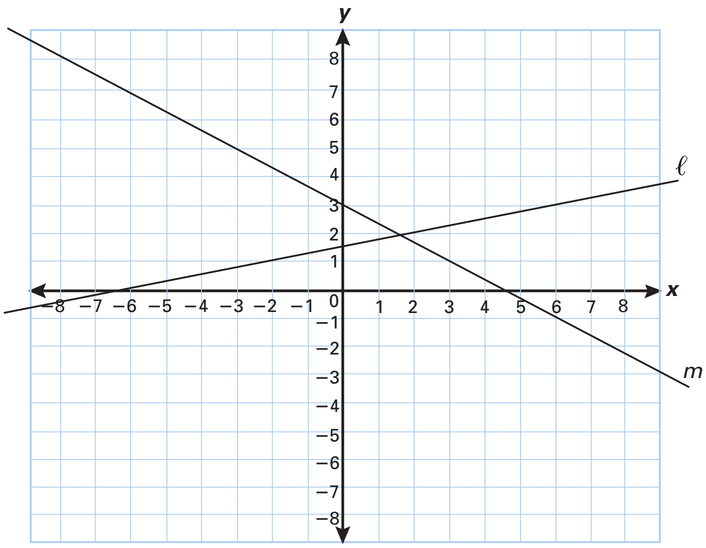
   b. $-5 + 3x = 16 - 4x$

2. Write a "frog problem" for each of the following equations. Then solve each equation.

   a. $4 + 3x = 19 + 2x$

   b. $-4 + 3x = -19 + 2x$

Section E Intersecting Lines



1. Which of the two lines shown on this graph has the equation $y = 1.5 + 0.25x$? Explain your answer.

2. What is the equation of the other line?

3. Find the point of intersection of the two lines.

4. Suppose you drew a line p that intersects line ℓ at (6, 3) and line m at (3, 1). What is the equation of line p?

Answers to Check Your Work

Section A Where There's Smoke

1. a. 210°

b. 310°, 130°

2. a. 5°, 10°

b. Tower A: 18°
Tower C: 135°

3. No, at least one report must be incorrect, because the lines going in the same direction are parallel. Parallel lines do not intersect.

4. a. $y = 3$

b. The inequality is $y < 3$.

5. The following lines are the boundaries of the region.

$y = -3$ and $y = 3$, so $-3 < y < 3$
$x = -4$ and $x = 4$, so $-4 < x < 4$

Section B Directions as Pairs of Numbers

1. a. $P(4, 3)$

b. The direction from O to P can be described with the direction pair [+4, +3] and the direction pair [+8, +6] or [+2, +1.5] or other pairs that have the same ratio as $\frac{+3}{+4}$.

c. Different points can be labeled, for example (−4, −1) and (−2, 0) and (2, 2). Note that all points must lie on a straight line through P and (−2, 0).

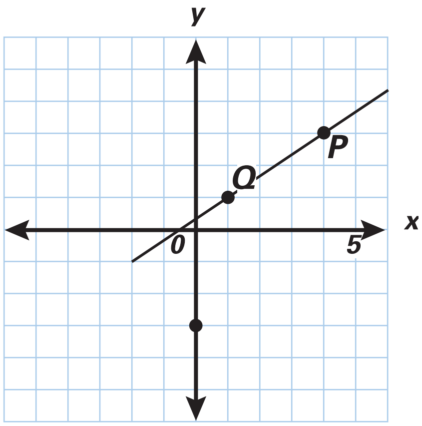
d. You can draw a line through P and any of the points mentioned in your answer to 1c.

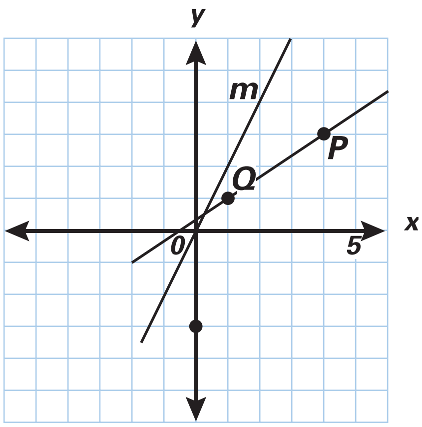
2. a. Different direction. The first pair moves right and up, and the second pair moves right and down.

b. Same direction. They both go to the right and up.

c. Same direction. They both go due east.

d. Same direction. They both go right and up.





3. a. See graph at upper left.

b. The direction from *P* to *Q* can be described with the direction pair [−3, −2].

c. The slope can be found by using the direction pair from part **b.** So the slope is $\frac{-2}{-3}$, which can be simplified to $\frac{2}{3}$.

4. See graph at lower left.

5. a. There is only one line through these two points.

b. To find the slope, you must first find the direction from one point to the other. From (1, 2) to (26, 52), you go 25 steps in a horizontal direction and 50 steps in a vertical direction. So the direction pair is [+25, +50]. The slope is $\frac{+50}{+25} = 2$. It may help you to make a sketch of the situation.

Section C An Equation of a Line

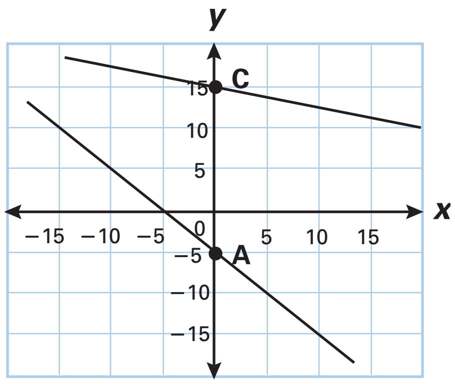
1. a. The *y*-intercept is −3 and the slope is 2.

b. If a line goes through (0, 0), the *y*-intercept is 0. So the equation is $y = 0 + 2x$ or even shorter $y = 2x$.

2. a. $y = 15 - \frac{1}{4}x$ or $y = 15 + (-\frac{1}{4})x$

b. $y = -5 - 1x$
or $y = -5 - x$

3.



You may have used different scales on the axes. If so, your lines will look different.

4. a. Many answers are possible; an example is $y = 2 - 3x$. When you draw a line with a positive y-intercept and a negative slope, it will always cross the y-axis above the origin, and it will run down to the right from there.

b. A line with y-intercept 0 goes through O (0, 0).

c. A line with slope 0 is a horizontal line since the vertical direction in the slope ratio must be 0.

Section D Solving Equations

1. Answers may vary. Sample answer:

A frog starts 7 dm from the path and takes three jumps of the same length. The expression to represent the diagram is $7 + 3x$.

2. a.

b.

$2 + 3x = 10 + x$ $-x$

$2 + 2x = 10$ -2

$2x = 8$ Divide by 2.

$x = 4$

3. You can use different methods to solve the equation, such as drawing frog diagrams, using a number line, or performing operations on both sides. Here is a sample solution using the method of performing operations:

$20 + 2v = 26 - 2v$

add $2v$ to both sides

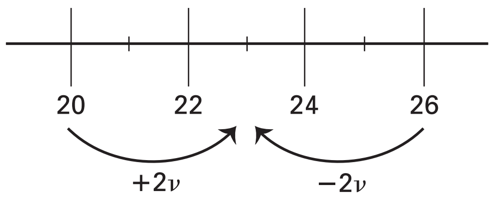
$20 + 4v = 26$

subtract 20 from both sides

$4v = 6$

divide both sides by 4

$v = 6/4 = 1.5$



4. a. $12 + u = 11 + 3u$
$12 = 11 + 2u$
$1 = 2u$
$u = 1 \div 2$
$0.5 = u$

b. $-4 + 2w = 2 + w$
$2w = 6 + w$
$w = 6$

c. $10 - v = 24 + v$
$10 = 24 + 2v$
$-14 = 2v$
$-14 \div 2 = v$
$-7 = v$

5. Problems will vary. Sample problems:

Easy:

$4 + 3x = 7 + 2x$
The frogs both jump in the same direction.

$4 + x = 7$
$x = 3$

Semi-difficult:

$4 + 3x = 19 - 2x$
One frog jumps in a positive direction, and one jumps in a negative direction.

$4 + 5x = 19$
$5x = 15$
$x = 3$

Difficult:

$-4 - 3x = -19 + 2x$

One frog jumps in a positive direction, and one jumps in a negative direction, left of 0.

$-4 = -19 + 5x$

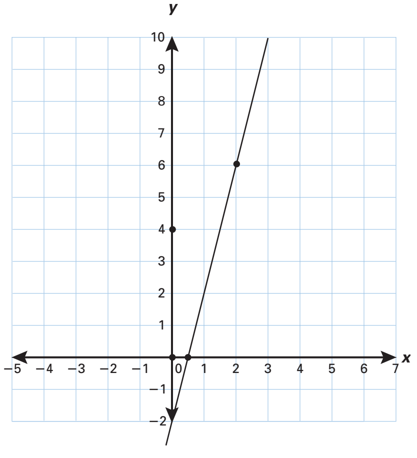
$15 = 5x$

$3 = x$

Section E Intersecting Lines

1. The point of intersection is $(-1, 3)$. This can be seen without a drawing because the x-coordinate must equal -1 and the y-coordinate must equal 3.

2. a.



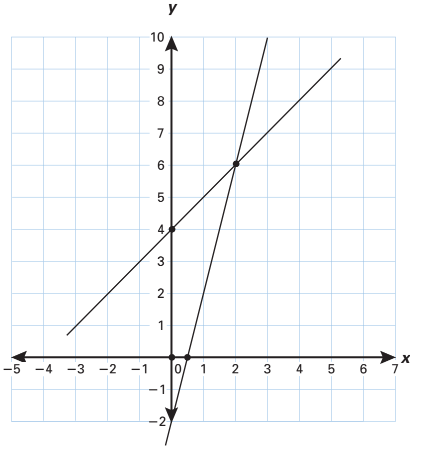
b. Any line with the same slope but a different y-intercept has no point of intersection with the given line, so the equation is $y =$ *any number* $+ 4x$.

c. The point of intersection is (2, 6).

You can be sure that (2, 6) is is correct by showing $x = 2$ and $y = 6$ fits both equations:

$6 = –2 + (4 \times 2)$

$6 = 4 + 2$



3. a. At the point of intersection, $x = -\frac{4}{5}$.

$-3 - 2x = 1 + 3x$

$-3 = 1 + 5x$

$-4 = 5x$

$x = -\frac{4}{5}$

b. At the point of intersection, $x = -\frac{7}{5}$.

$y = 1 + 3\left(-\frac{4}{5}\right)$

$y = \frac{5}{5} + -\frac{12}{5} = -\frac{7}{5}$

4. Line *j* goes through points (0,3) and (5, -5), so the slope is $\frac{-8}{5}$.

Line *k* goes through points (-3,4) and (0,-1), so the slope is $\frac{-5}{3}$.

The equation of line *j* is $y = 3 - \frac{8}{5}x$.

The equation of line *k* is $y = -1 - \frac{5}{3}x$.

The point of intersection is (–60, 99). Sample strategy:

$3 - \frac{8}{5}x = -1 - \frac{5}{3}x$	add $\frac{8}{5}x$ to both sides
$3 = -1 + (\frac{8}{5} - \frac{5}{3})x$	add 1 to both sides
$4 = -\frac{1}{15}x$	multiply both sides by -15

$x = -60$

$y = 3 - \frac{8}{5}(-60)$

$y = 3 - (-96) = 99$